Nuclear Theory - Course 127

NEUTRON BALANCE AND THE FOUR FACTOR FORMULA

When a reactor is operating at steady power, the chain reaction is just being maintained. One neutron only is available from each fission to cause a further fission. The reactor is then said to be just critical, with the neutron population remaining constant.

If neutron losses, by leakage or radiation capture, are reduced further, then more neutrons are available to cause fission. The number of fissions occurring in any one generation will be greater than in the previous generation. There is, therefore, a multiplication of neutrons. The multiplication factor, k, is defined by:-

$$k = \frac{\text{Number of neutrons causing fission in any one generation}}{\text{Number of neutrons causing fission in the previous generation}}$$

Consideration will now be given to the quantities or factors on which k depends. In this lesson the factors themselves will be introduced by discussing the neutron balance in a reactor operating at steady power.

The Neutron Cycle

The neutron from each fission, that causes a further fission to maintain the chain reaction, goes through a typical cycle which is shown in Fig. 1.

Fast Neutron born or produced at fission	→	Fast Neutron slowed down in moderator to become a thermal neutron	→	Thermal neutron captured in U-235 to cause fission

Of the $2\frac{1}{2}$ neutrons produced at fission, the only one that goes through this cycle is the one that is used to maintain the chain reaction. The others are lost by capture or escape during this cycle. It is useful to know when and how these neutrons are lost since these losses affect the value of k. A complete neutron cycle will, therefore, be considered. This cycle shown in Fig. 2, is very similar to the one given in the Level 2 course except that symbols are used instead of numbers.

We start the cycle with N thermal neutrons in the reactor, and go round the cycle step by step as follows. The continuous lines indicate steps that contribute to the chain reaction and the dotted lines indicate losses of neutrons.

(a) A fraction f, of the thermal neutrons, are assumed to be absorbed in the fuel and a fraction (1 - f) is, therefore, lost by radiative capture in material other than fuel. So fN thermal neutrons are absorbed in the fuel.

The quantity f is called the THERMAL UTILIZATION FACTOR. It could also be defined as the ratio of the neutrons absorbed by the fuel to the total neutrons absorbed in the reactor.

(b) Not all the neutrons absorbed by the fuel will cause fission. Some will be lost by radiative capture. Let a fraction "a" cause fission in U-235, the remainder, (1 - a), being lost by radiative capture. Therefore, afN fissions will occur.

(c) Each fission will produce ν (Greek letter nu) fast neutrons. ν is about 2.5 for U-235. Therefore, a νfN fast neutrons are produced by the original N thermal neutrons.

The product aν is usually replaced by one letter η (Greek letter eta) so that ηfN fast neutrons are produced by U-235 fissions. The factor η now represents the number of fast neutrons produced from each neutron captured in the fuel, whereas ν represents the number of neutrons produced from each neutron actually causing fission.

(d) In (c) we have only considered fast neutrons produced from U-235 fissions, caused by thermal neutron capture. Some of these fast neutrons produced by U-235 fissions, cause U-238 fissions before they are slowed down. Since each U-238 fission produces 2½ new neutrons, the total fast neutron production will be greater than ηfN. This additional contribution is shown by increasing the fast neutron production by a factor ϵ (Greek letter epsilon) ie, the total fast neutrons produced by both U-235 and U-238 fission is $\epsilon\eta$fN.

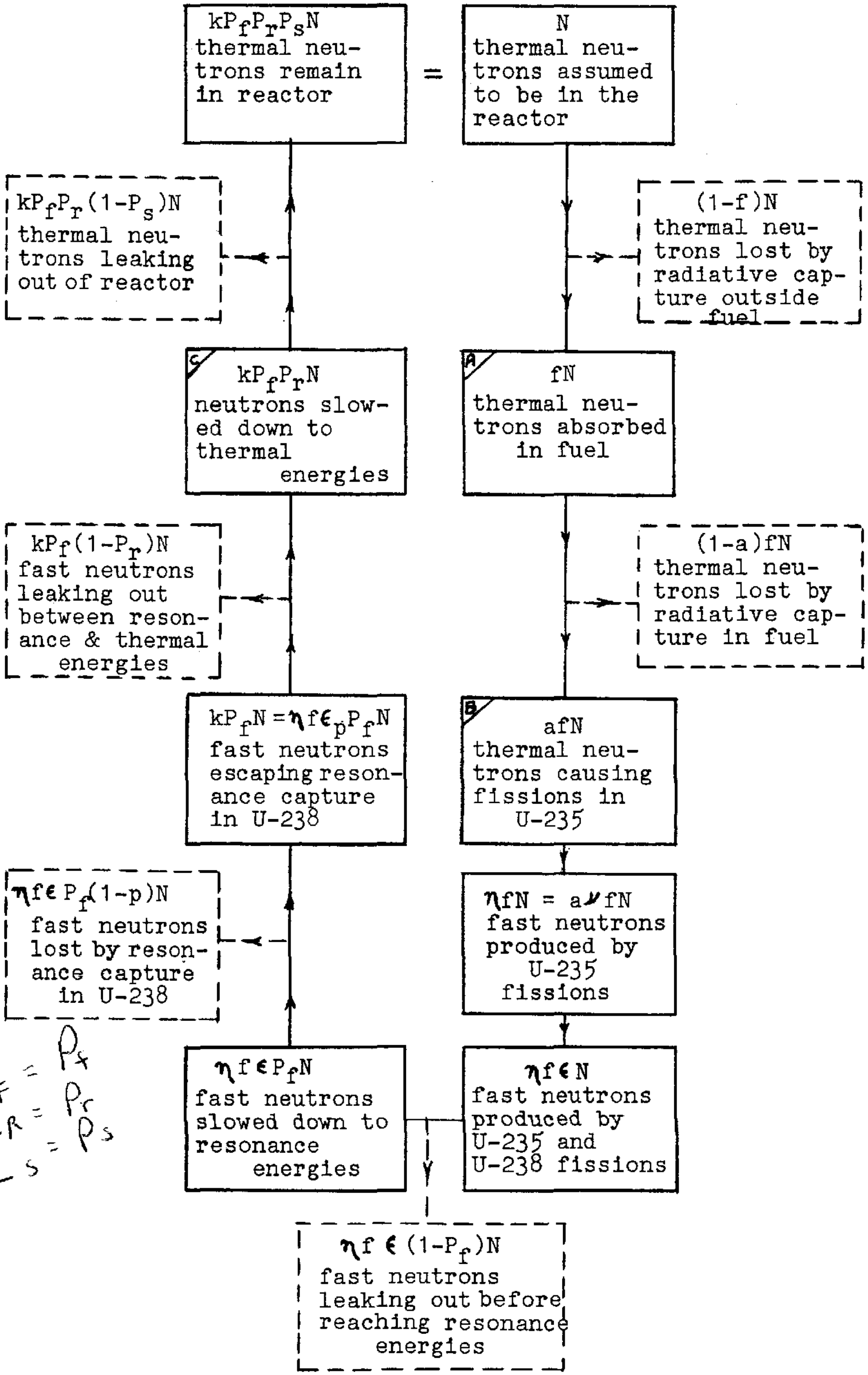
ϵ is known as the FAST FISSION FACTOR.

(e) The fast neutrons produced now have to be thermalized. Some of them escape from the reactor before the resonance energies are reached. If $\epsilon\eta f(1-P_f)N$ escape, the remainder $\epsilon\eta fP_fN$ remain and are slowed down to resonance energies.

P_f, is therefore the fraction of the fast neutrons produced which reach resonance energies without escaping.

(f) If a fraction p now avoids resonance capture, $\epsilon\eta f\rho P_fN$ neutron are slowed down below resonance energies and the remainder are lost by resonance capture.

Fig. 2 THE NEUTRON CYCLE



$\Lambda_F = P_f$

$\Lambda_R = P_r$

$\Lambda_S = P_s$

(f) This fraction, p, is known as the RESONANCE ESCAPE PROBABILITY.

Neglecting the escaping neutrons we have now allowed $\epsilon\eta fpN$ neutrons to slow down to thermal energies and there were N thermal neutrons originally. So the neutrons have multiplied $\epsilon\eta fp$ times, ie, the neutron multiplication factor, ignoring leakage, is $\epsilon\eta fp$. Thus $k = \epsilon\eta fp$ and kP_fN neutrons slow down below resonance energies.

(g) More neutrons will however escape before thermal energies are reached and only some fraction P_r will become thermalized. Therefore, kP_fP_rN thermal neutrons are produced.

(h) Some thermal neutrons also escape and $kP_fP_rP_sN$ neutron remain in the reactor, P_s being the fraction of the thermal neutrons that do not escape.

The product P_fP_r represents the total fraction of the fast neutrons, which are produced at fission and which do not escape, ie, P_fP_r is the total *fast neutron non-leakage probability*.

It is not necessary to start the cycle at the particular box chosen. It could be started from the boxes marked A, B or C, for instance. However, if the cycle is to be continuous and the chain reaction just maintained, then on completing the cycle back to the starting box, the same number of neutrons must appear in the box as was there when the cycle was started.

For example a simplified cycle, using numerical values and starting at box A, would be as shown in Fig. 3.

Of 100 thermal neutrons absorbed in U-235 nuclei, 17 suffer radiative capture and the remaining 83 cause fission. From 83 U-235 fissions, 210 fast neutrons are produced, (ie, $\nu = 2.54$). The U-238 fissions are then ignored, since $\epsilon \approx 1$. Nine fast neutrons then leak out of the reactor so that:

$$P_fP_r = \frac{201}{210} \approx 0.96$$

16 fast neutrons suffer resonance capture and $p = \dfrac{185}{201} = 0.92$.

Thus, 185 thermal neutrons are produced by slowing down, in box C. Of these 10 escape, (ie, $P_s = 175/185 \approx 0.95$), leaving 175. A further 19 thermal neutrons suffer radiative capture outside the fuel, so that:

$$f = \frac{(185 - 10 - 19)}{(185 - 10)} = \frac{156}{175} = 0.89$$

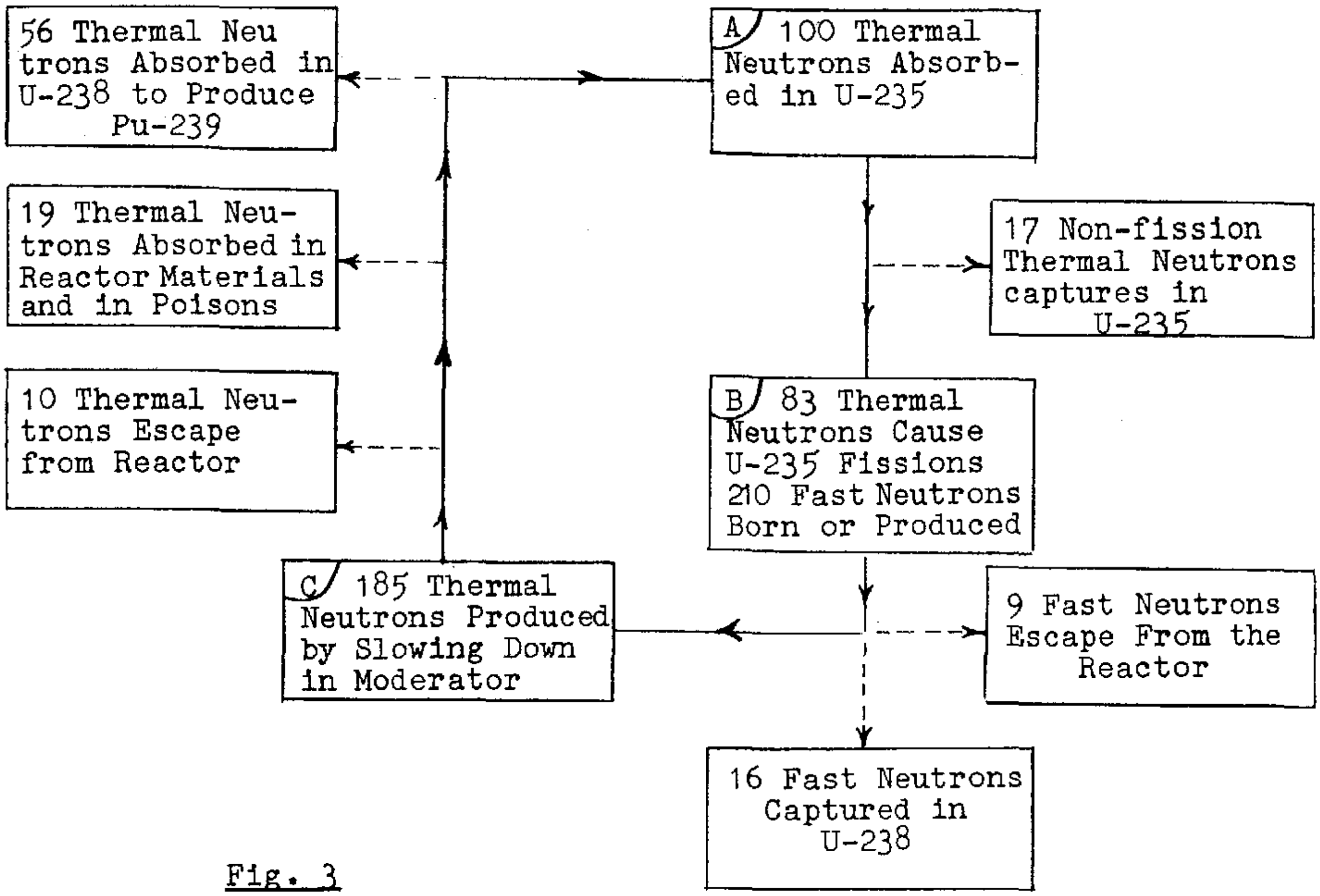


Fig. 3

Finally 56 neutrons suffer radiative capture in U-238, so that the total neutron radiative capture losses in the fuel is 56 + 17.

$$\text{Hence } a = \frac{156 - 73}{156} = \frac{83}{156} = 0.53$$

The value of "a" should, of course, be equal to σ_f / σ_a for natural uranium. Thus, since $\sigma_a = 7.6$ barns and $\sigma_f = 4$ barns, this ratio should be 0.53.

$$\eta = \frac{210}{156} = 1.3$$

$$\text{also } \eta = a\nu = 2.54 \times 0.53 = 1.35$$

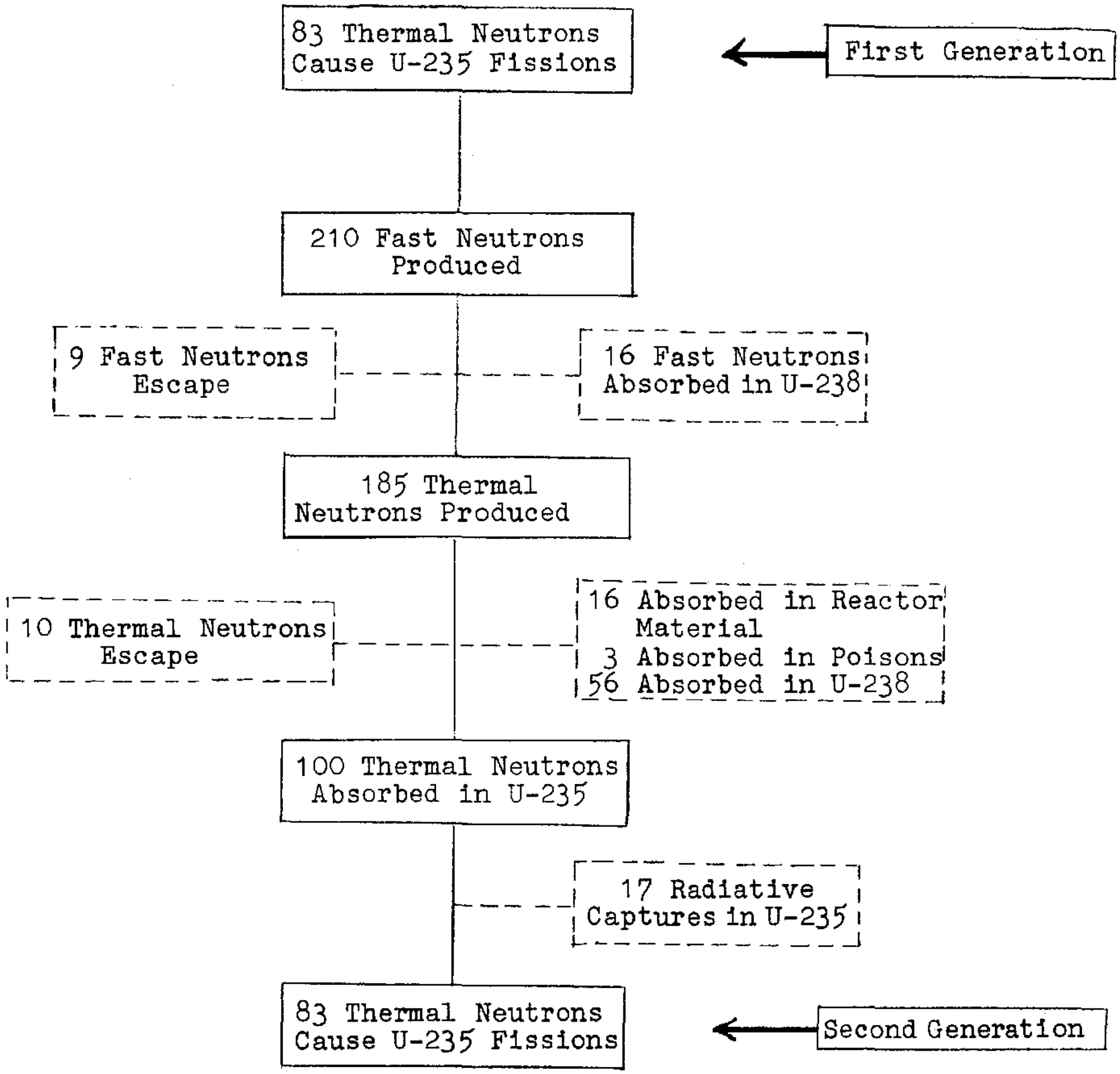


Fig. 4

Alternatively, the cycle could have been started from box B. Fig. 4 shows such a cycle, starting from box B. The cycle has been written down in a somewhat different manner to illustrate the significance of successive generations of neutrons.

Again, fast fissions in U-238 have been ignored to simplify the cycle. The 19 thermal neutrons, lost by radiative capture outside the fuel material have now been subdivided into 16 captured in reactor materials and 3 captured by poisons. The poisons, such as Xe-135, are, of course, produced in the fuel elements but they do not form a part of the fissile or fertile material which are classified as fuel material.

At the end of a further cycle there would be a third generation of neutrons and so on. The number of neutrons generated in each generation is not important. If at the end of each cycle they remain constant, the chain reaction is just being maintained. If the number of fissions in succeeding generations decreases, as would be the case if the neutron losses increased, then the chain reaction cannot be maintained and the reactor power decreases. If the number of neutrons in succeeding generations increases, neutron multiplication occurs and the reactor power increases.

The Four Factor Formula

Returning to the cycle in Fig. 2, the number of neutrons after each cyle will remain constant and the chain reaction will just be maintained if: -

$$kP_fP_rP_sN = N$$

$$\text{or} \quad kP_fP_rP_s = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

This is, then, the condition for criticality

It is much easier, when considering the factors that affect k, to initially ignore all neutron leakage out of the reactor and then allow for it later. Theoretically, the leakage is zero only for a reactor of infinite size but, in practice, it is near enough zero for large reactors. To indicate that an infinitely large system is being considering and that neutron leakage is being ignored, the multiplication factor is written as k_∞ . The condition for criticality, with the chain reaction just being maintained now becomes:

$$k_\infty = \eta \epsilon pf = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

This equation is frequently referred to as the FOUR FACTOR FORMULA. It connects the neutron multiplication factor, k_∞ , with the four factors, η , ϵ , p and f, which determine its value.

If a system of finite size is being considered, the effective multiplication factor, k_e is given by:

$$k_e = k_\infty - \text{neutron leakage}$$

$$\text{or} \quad k_e = \eta \epsilon pf - \text{neutron leakage} \quad \dots\dots\dots\dots\dots\dots (3)$$

The condition for criticality is, then:

$$k_e = 1 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (4)$$

Equations (1) and (4) specify the same condition.

Non-leakage Probabilities

P_f is the fraction of fast neutrons that are reduced to resonance energies without escaping out of the reactor. P_r is the fraction of neutrons which are slowed down from resonance to thermal energies without escaping. The product P_fP_r, therefore, represents the total fraction of fast neutrons that do not escape, ie, P_fP_r is the fast neutron non-leakage probability.

$$\text{Now } P_fP_r = e^{-B^2L_s^2}$$

where B is a quantity called the BUCKLING, which is associated with the flux distribution and L_s is the slowing down length of fast neutrons. This equation shows how L_s determines the fast neutron leakage out of a reactor. The longer the slowing down length the smaller P_fP_r and the greater the leakage. This is another reason why the thermalization should require as few collisions as possible.

P_s is the fraction of thermal neutrons that stay in the reactor, ie, the thermal neutron non-leakage probability.

$$\text{Now } P_s = \frac{1}{1 + B^2L^2}$$

where L is the diffusion length. If L increases, P_s decreases and thermal neutron leakage increases. So the diffusion length of thermal neutrons should be small.

Equation (1) now becomes: –

$$\frac{k_\infty \; e^{-B^2 L_s^2}}{1 + B^2 L^2} = 1 \quad \text{.........................(5)}$$

as the condition for criticality.

Equation (3) can also be more accurately written as

$$k_e = \frac{k_\infty \; e^{-B^2 L_s^2}}{1 + B^2 L^2} = \frac{\eta \epsilon p f \; e^{-B^2 L_s^2}}{1 + B^2 L^2} \quad \text{........(6)}$$

ASSIGNMENT

1. Define the neutron multiplication factor, k.

2. (a) In what ways can the fast neutron, produced at fission, be lost while being thermalized?

   (b) By what methods, other than leakage, are thermalized neutrons lost so that they are not available for fission?

3. Define or explain the terms "Thermal Utilization Factor", "Fast Fission Factor" and "Resonance Escape Probability".

4. (a) What expression expresses the condition for criticality if leakage is considered?

   (b) Write down the "Four Factor Formula" and explain how it is obtained from the expression in 4a.

5. Explain why the slowing down length and diffusion length of a neutron affect neutron leakage.

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